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Seiberg Duality in Matrix Model

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ABSTRACT: In this paper, we use the matrix model to show the Seiberg duality in the case of complete mass deformation.

KEYWORDS: Seiberg Duality, Matrix Model.

Motivation Since Dijkgraaf and Vafa gave the matrix model conjecture [1, 2, 3], there are a lot of related works to check, prove and generalize this conjecture, see [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25]. Especially in [15] and [23], a field theory argument has been given to show why the calculation of exact lower energy superpotential can be reduced to integration in the corresponding matrix model.

The matrix model conjecture holds for more general cases although the primary focus is still the theory with adjoint fields. The generalization to fundamental fields has been discussed in [14, 13, 16, 18, 19]. These kinds of generalization are desired because most applications in field theory will have fundamental matters. One particular interesting application is to see the Seiberg duality [26] in the matrix model.

The standard example of Seiberg dual pair is following. At one side, we have electric theory $SU(N_c)$ with N_f flavors $Q_j, \widetilde{Q}_j$ and no superpotential. At another side, we have magnetic theory $SU(N_f - N_c)$ with N_f flavors $q_j, \widetilde{q}_j$, meson field X_i^j and superpotential

$$W_{mag} = \frac{1}{\mu} X_i^j q_j \widetilde{q}^i. \tag{1}$$

where μ is a dynamical scale [27] (equation (3.125)). In general case, these two theories will have flat directions in the moduli space, so when we try to do the matrix model integration, we must take care of the zero modes as did by Berenstein[11]. To avoid the complexity, we can deform above theories by adding mass terms. For example, in the electric theory we add superpotential with non-degenerated mass matrix[1]

$$W_{elec} = \sum_{j=1}^{N_f} Q_j m_j \widetilde{Q}^j \tag{2}$$

Since we give all flavors nonzero mass in the electric theory, the IR field theory will be pure super Yang-Mills theory and we know the exact effective action as

$$W_{YM} = N_c(\hat{\Lambda}^{3N_c})^{\frac{1}{N_c}} \tag{3}$$

where $\hat{\Lambda}$ is the dynamical scale at IR and related to dynamical scale Λ at UV by

$$\hat{\Lambda}^{3N_c} = det(m)\Lambda^{3N_c - N_f} \tag{4}$$

The deformation in the electric theory will induce the corresponding deformation in magnetic theory as

$$W_{mag} = \frac{1}{\mu} X_i^j q_j \widetilde{q}^i + tr(Xm) \tag{5}$$

[1]We can always redefine the field to bring the mass matrix into diagonal form.

Matrix integration Now we will do the matrix integration for both electric and magnetic theories. Let us do the electric theory first. The matrix model is

$$\frac{1}{Vol(U(N))}\int\prod_{j=1}^{N_f}dQ_jdQ_j^\dagger e^{-\frac{1}{g_s}\sum_{j=1}^{N_f}Q_jm_jQ^{\dagger j}} \tag{6}$$

where every Q is a M component vector and $\widetilde{Q}$ in the field theory has been treated as a conjugate of Q [2]. The part of flavor integration is

$$\int\prod_{j=1}^{N_f}dQ_jdQ_j^\dagger e^{-\frac{1}{g_s}\sum_{j=1}^{N_f}Q_jm_iQ_j^\dagger}=\prod_{j=1}^{N_f}(\frac{\pi g_s}{m_j})^M$$
$$=(\pi g_s)^{MN_f}(det(m))^{-M}=e^{MN_f\log(\pi g_s)-M\log(det(m))}$$

Except that, there is also the volume factor $e^{\frac{M^2}{2}\log M}$. Adding everything together we get

$$\begin{aligned}&exp(\frac{M^2}{2}\log M+MN_f\log(\pi g_s)-M\log(det(m))\\&=exp(\frac{1}{g_s^2}\frac{S^2}{2}\log\frac{S}{g_s}+\frac{1}{g_s}[SN_f\log(\pi g_s)-S\log(det(m))])\\&\equiv exp(\frac{1}{g_s^2}\mathcal{F}_{\chi=2}+\frac{1}{g_s}\mathcal{F}_{\chi=1})\end{aligned}$$

where we have grouped all term according to the genus expansion. From the corresponds given in [3, 13], we should identify

$$-W_{elec}=N_c\frac{\partial(\frac{S^2}{2}\log\frac{S}{g_s})}{\partial S}+[SN_f\log(\pi g_s)-S\log(det(m))] \tag{7}$$

In the matrix model, S,M are dimensionless number, but in field theory, S is dimension 3. To match the field theory result, we need to replace g_s by some proper dimensional number heuristically. For example, we need to replace $\frac{S}{g_s}$ to $\frac{S}{\Lambda^3e^{3/2}}$ to reproduce the well known Veneziano-Yankielowicz term. Same heuristic argument tell us that πg_s in the second term must be replaced by dimension one number which can be chosen naturally as Λ. Doing these replacement we get

$$\begin{aligned}-W_{elec}&=N_c\frac{\partial(\frac{S^2}{2}\log\frac{S}{\Lambda^3e^{3/2}})}{\partial S}+[SN_f\log(\Lambda)-S\log(det(m))]\\&=N_c[S\log\frac{S}{\Lambda^3}-S]+[SN_f\log(\Lambda)-S\log(det(m))]\\&=N_c[S\log(\frac{S}{(\Lambda^{3N_c-N_f}det(m))^{\frac{1}{N_c}}})-S]\end{aligned}$$

Minimized it we get $S=(\Lambda^{3N_c-N_f}det(m))^{\frac{1}{N_c}}$ and

$$W_{elec}=N_c(\hat{\Lambda}^{3N_c})^{\frac{1}{N_c}}=N_c(\Lambda^{3N_c-N_f}det(m))^{\frac{1}{N_c}} \tag{8}$$

which is the famous Affleck-Dine-Seiberg superpotential for pure $U(N)$ Yang-Mills gauge theory. Notice that the equation (4) is naturally shown in the matrix model. Furthermore, in our calculation, we keep N_f fixed while taking $M \to \infty$.

Now we do the matrix model integration for the magnetic theory

$$\frac{1}{Vol(U(N))}\int dX \prod_j dq_j dq_j^\dagger exp(\frac{-1}{g_s}[tr(mX)+\sum_{i,j=1}^{N_f}\frac{1}{\mu}X_i^j q_j q^{\dagger i}]) \tag{9}$$

where X is $N_f \times N_f$ matrix and q is N component vector. Doing the X integration first, we get a delta-function

$$\delta(m+\frac{q_j q^{\dagger i}}{\mu}) = \mu^{N_f^2}\delta(\mu m + q_j q^{\dagger i}) \tag{10}$$

Now the remainder part is exact the integration given in [19] (equation (6) with $-\mu m$ taking the place of X there. Furthermore, since we always take large N limit with fixed N_f, there is not problem to apply their result.), so we just cite their result

$$W_{mag} = (\widetilde{N}_c - N_f)(\frac{\widetilde{\Lambda}^{3\widetilde{N}_c - N_f}}{det(-\mu m)})^{\frac{1}{\widetilde{N}_c - N_f}} \tag{11}$$

where we use tilde to emphasize that it is in the magnetic theory. Using the result $\widetilde{N}_c - N_f = -N_c, \quad det(-\mu m) = (-)^{N_f}\mu^{N_f} det(m)$ and

$$\Lambda^{3N_c - N_f}\widetilde{\Lambda}^{3\widetilde{N}_c - N_f} = (-)^{N_f - N_c}\mu^{N_f}, \tag{12}$$

which can be found, for example, in [27]. we get immediately

$$W_{mag} = N_c(\Lambda^{3N_c - N_f} det(m))^{\frac{1}{N_c}} \tag{13}$$

where the $-$ sign in front of (11) is canceled exactly by factor $(-)^{N_c}$. Comparing (8) and (13) we see that they are same. Thus we give an example to show how the matrix model can see the Seiberg duality.

We must emphasize that we did not realy derive the Seiberg duality from the matrix model because the Seiberg duality holds in general situation even without any mass deformation. We feel that to address the full Seiberg duality, we must understand how to do the matrix integration when there is flat directions in moduli space along the line [11]. Obvious, work should be generalized other generalized Seiberg duality, for example, the toric duality addressed in [28, 29, 30, 31, 32, 33, 34].

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